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## ABSTRACT

When a soft mechanical metamaterial, consisting of a regular array of representative volume elements (RVEs), is stressed up to a large strain, the delicately tailored behavior of the RVE does not prevail in the metamaterial due to the boundary effect and manufacturing imperfections. A metamaterial sheet comprising RVEs designed for snapping-back behavior exhibits random snapping-through instability when uniaxially stretched. We conceptualize that loss of representativeness of RVE can be avoided by introducing fiber reinforcement to regulate boundary conditions. Through a combination of experiments and numerical simulation, we demonstrate that fiber reinforcements tune behavior of a metamaterial sheet from random snapping-through to sequential and even selective snapping-back instability by introducing small structural variations. Ideal snapping-back instability, characterized by sharp variations of forces in both loading and unloading processes, is captured, while the latter is typically hard to observe in real experiments. Enhanced energy dissipation rate from 25.3% for the case without fiber to 46.4% for the case with fiber-reinforcement is recorded in experiments, when the metamaterial sheet is stretched up to 200% and then released to restore its original length.



## I. INTRODUCTION

Mechanical metamaterials are man-made materials comprised of an array of regular/irregular arrangement of unit cells, and their unusual or novel properties originate from design and patterning of unit cells rather than from chemistry of their constituents. Soft mechanical metamaterials are made of elastomeric materials, mainly readily fabricated via 3D printing or molding, whose behaviors are dictated by deformation, stress, and motion.[1–6] Soft mechanical metamaterials have attracted increasing attention in recent years, because of their diverse potential applications in soft robotics,[7,8] deployable and morphing structures,[9–11] reusable energy dissipation,[12–15] and flexible electronics.[16]

One particular class of soft mechanical metamaterial is cellular three-dimensional lattices or two-dimensional perforated sheets, featured by the existence of a large number of slender structural elements, including inclined rods, curved beams, shallow arches, and hinge-like nodes. They can be fabricated by 3D printing, molding, or simply by laser cutting. Subjected to external loadings, the slender elements undergo nonlinear instability, buckle, or snap to new equilibrium or rotate like a hinge, which could be leveraged to achieve new functions such as sequential buckling, multistability, energy trapping, and hysteresis.[17,18] Focusing on the cut-mediated metamaterial sheets with thickness from hundreds of micrometers to several millimeters and with modulus about tens of megapascal, they mainly work as structures that only sustain tensile loading and are prone to large deformation and instability.

## II. INSTABILITY IN A MECHANICAL METAMATERIAL SHEET UNDER TENSION

Two types of instability, recognized as snapping-through and snapping-back, are commonly observed in experiments or numerical simulations of the metamaterial sheets. The difference between the force–displacement curves of two systems undergoing snapping-through and snapping-back instability is subtle.[19] As shown in Fig. 1, for the snapping-through case, the force–displacement curve has only force limit points; snapping occurs at force limit points only when the system is loaded in a force-controlled way, and no snapping occurs in a displacement-controlled scenario and the whole equilibrium path is physically accessible. For the snapping-back case, on the other hand, the force–displacement curve typically has both force and displacement limit points. Force limit points cause force-controlled structures to snap to another stable equilibrium, whereas displacement limit points cause displacement-controlled structures to jump.[20] In this case, a branch of equilibrium path is unstable and physically inaccessible, and it can only be captured numerically by using the algorithm such as arc-length continuation. Concerned with the slope of the force–displacement curves, there exists a negative slope region for the snapping-through case. For a displacement-controlled structure undergoing snapping-back, the physically inaccessible equilibrium path exhibits a positive slope, and both force and displacement decrease beyond the displacement limit points. In real experiments, the difference and transition between snapping-through and snapping-back instability is obscure. A sharp change of force is observed in a displacement-controlled experiment when snapping-back instability occurs: the reaction force snaps downward and decreases sharply in the loading process but snaps backward and increases steeply in the unloading process, while displacement is conserved when snapping or jump occurs.

When a soft mechanical metamaterial sheet consisting of finite representative volume elements (RVEs) is uniaxially stretched,

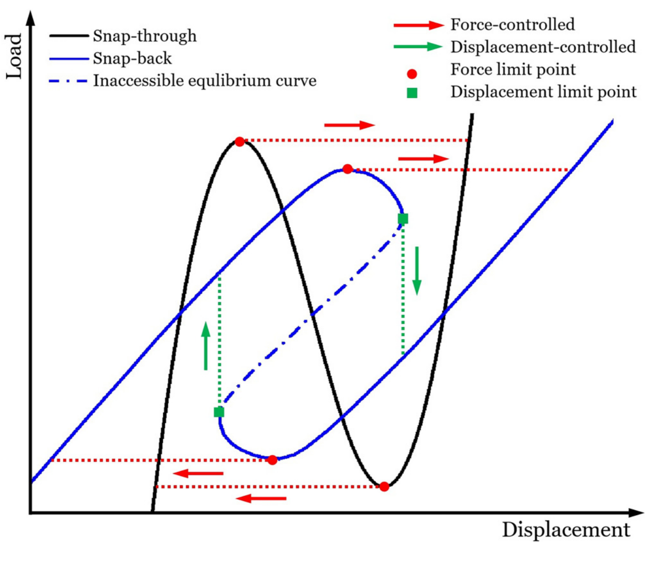


**FIG. 1.** Schematic force–displacement curves for a system undergoing snapping-through (solid black line) and snapping-back (solid blue line) instability. Jumps may occur at limit points either in a force-controlled or displacement-controlled case and are indicated by the arrows.

the resulting large deformation and the associated instability for different RVEs are in general inhomogeneous. Inhomogeneous deformation originates from geometric nonlinearity, boundary effect, and sensitivity of buckling instability to various imperfections. Thus far, this type of finite size soft mechanical metamaterials often exhibits random instability.[13,15] It is highly desirable to tune mechanical behavior of such soft mechanical metamaterials from random to deterministic, and even programmable. This is crucial for applications where controllable instability is desperately needed for their functions, but it is not well studied in the context of soft mechanical metamaterials. We demonstrate a concept of achieving selective snapping-back instability in a finite size metamaterial sheet, in which small structural variations single out layers that snap selectively under external loading. This was realized by embedding hard fibers into a soft metamaterial sheet. The fiber reinforcements modify boundary conditions of a uniaxially stretched sheet, eliminate necking for large deformation, and change the deformation mode from uniaxial tension to pure shear. A programmable snapping-back instability was triggered in a layer-by-layer fashion, in contrast to random snapping-through instability in a finite size structure without fibers. In addition, a more sudden and harsh movement was attained in the case of snapping-back instability as compared to the snapping-through counterpart, thus more kinetic energy was dissipated and an enhanced hysteresis was achieved within a loading–unloading cycle.

## III. RESULTS AND DISCUSSIONS

Figure 2 presents metamaterials comprising $5 \times 5$ RVEs with dimensions given in the insets. Figure 2(a) for a homogeneous metamaterial without fibers, and Fig. 2(b) for a bi-material sheet with fiber reinforcement which is marked in yellow. The homogeneous metamaterial and its RVE was modified a little bit from our previous design,[15] in which snapping-back instability was attained for a typical RVE analysis with periodic boundary conditions. The inset in Fig. 2(b) also gives the fabrication process of the fiber-reinforced metamaterial sheet. The soft metamaterial sheet was fabricated of polyurethane rubber material (HEI-CAST 8400, H&K Ltd, Tokyo, Japan) using a molding process and the hard fibers made of Nylon 12 were produced by a HP Jet Fusion 3D 4210/4200 printer (Hewlett Packard, Barcelona, Spain). The fibers and rubber were then bonded together with 502 glue to make a bi-material sheet. Note that the out-of-plane thickness of all sheets was fixed as 4 mm for all samples.

Finite element (FE) simulation was carried out using the commercial code ABAQUS. ABAQUS/standard for implicit dynamics was adopted to model the quasi-static process. The rubber material behavior was effectively captured using a hyperelastic incompressible Neo-Hookean material model with initial shear modulus $G = 0.34$ MPa (Young's modulus $E = 1$ MPa) and Poisson's ratio $\mu = 0.49$ (see Fig. S1 in the supplementary material for details) and the fibers were modeled as a linear elastic material with Young's modulus $E = 2$ GPa and Poisson's ratio $\mu = 0.35$. Approximately 40 000 6-node quadratic plane stress triangle (CPS6) elements with a minimum size of 0.3 mm was adopted after a mesh independence check.

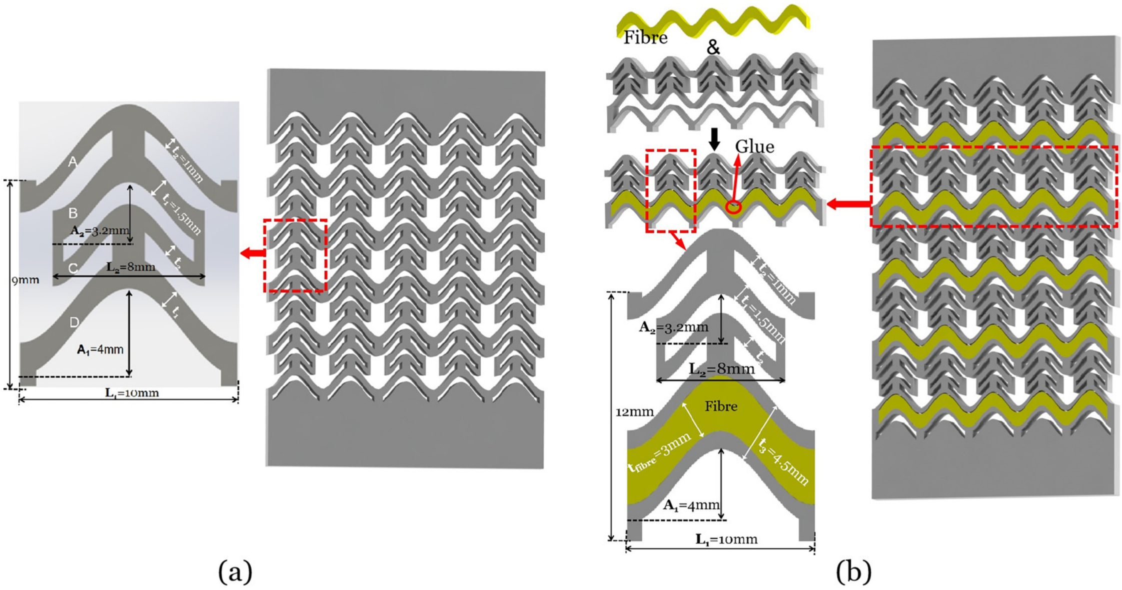


FIG. 2. A homogeneous mechanical metamaterial (a) and a bi-material metamaterial with fiber reinforcement (b) and their corresponding RVE. Also included in (b) is the fabrication process of the fiber reinforced mechanical metamaterial.

Figure 3 shows the loading–unloading curves and snapshots of deformation obtained from both experiments and simulation for a homogeneous metamaterial without fibers. Figure 3(a) plots the stress, which was normalized by Young's modulus of the rubber material, as a function of the applied tensile strain, with the black solid curve for experiments and the red dotted line for numerical simulation. Though the simulation captures the overall trends of the stress–strain curve observed in experiments, a noticeable discrepancy still exists, either in terms of wavy shape of stress–strain curve or the strains for the complement of instability. This discrepancy is attributed to the intrinsic complexity and randomness of the occurrence of snapping-through instability in this particular metamaterial sheet. It is observed from both experiments and FE simulation that, under uniaxial tension, the RVEs in our homogeneous metamaterial sheet snap in a random manner in terms of both number and position (see Movie S1 in the supplementary material), which essentially makes our metamaterial sheet different from the previously reported metamaterials.[13] Figures 3(b) and 3(c) give the snapshots obtained from experiments and FE simulation. Points ① to ⑥ in Figs. 3(a) and 3(b) as well as points I to VI in Figs. 3(a) and 3(c) indicate the valleys in the wavy loading curves, at which snapping-through or snapping-back instability completes. In Figs. 3(b) and 3(c), the inserted strains $\epsilon_E$ and $\epsilon_S$ represent strains corresponding to points ① to ⑥ and points I to VI recorded in experiments and simulation, respectively. The segments undergoing instability in Figs. 3(b) and 3(c) are marked in purple for better view. The number and positions of the purple segments in Figs. 3(b) and 3(c) imply that the occurrence of instability is rather random. Note that points ① to ⑤ in the experimental curve in Fig. 3(a) are identified as the ends of force dropping regions associated with snapping-through processes, while only the sudden drop associated with point ⑥ corresponds to a snapping-back process, in which sharp decrease of force is observed. No obvious snapping backward occurs following the whole unloading path.

Figure 4 presents the results for a metamaterial sheet reinforced by fibers, which are marked in orange in Figs. 4(b) and 4(c). Figure 4(a) plots the normalized stress–strain curves from experiments (see Movie S2 in the supplementary material) and FE simulation, which are represented by black solid line and red dashed line, respectively. Points ① to ⑥ as well as points I to VI also indicate the valleys of the wavy stress–strain curves. Figures 4(b) and 4(c) show the snapshots obtained from experiments and simulation, also with corresponding strains inserted for comparison. In big contrast to the random occurrence of snapping-through instability in local segments of a homogeneous sheet given in Fig. 3, the purple lines undergoing instability in Fig. 4 exhibit a sequential layer-by-layer fashion, though the order of sequence is still random. Much better agreement between simulation and experiments is achieved for the same set of material parameters. Both loading and unloading processes exhibit sequential snapping-back, and each peak and sharp change in the stress–strain curves denotes the occurrence of instability. It should be pointed out that it is hard to achieve ideal snapping backward in the unloading process. This is verified from the results in Fig. 3(a) and also proved by a recent experimental result on a wide hyperelastic column.[19] Nevertheless, ideal snapping downward as well as snapping backward are demonstrated in Fig. 4(a) for the loading and unloading processes, respectively. The difference between the results in Figs. 3 and 4 is interpreted as follows. If a piece of soft elastomeric sheets is fixed at two ends and stretched uniaxially in the vertical direction, the two layers on the top and bottom boundaries and

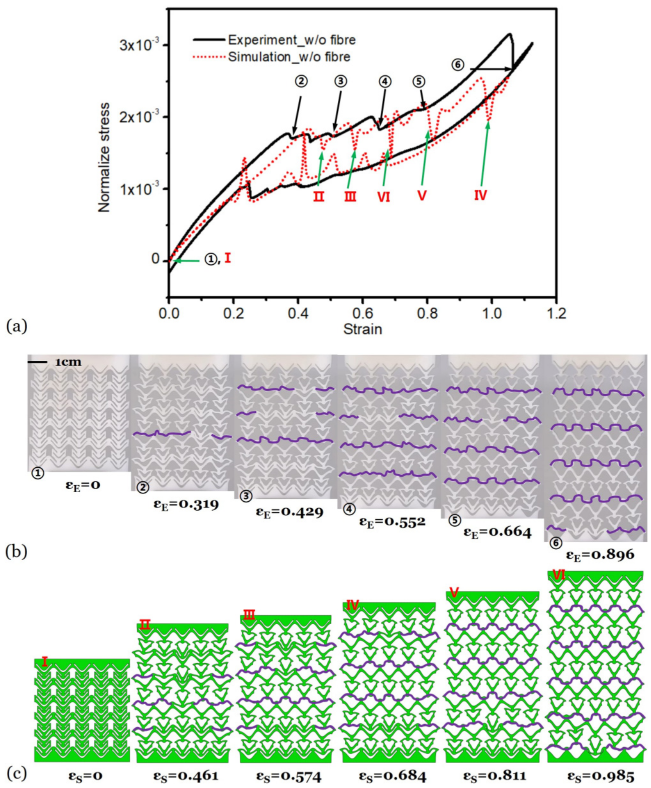


**FIG. 3.** A homogeneous metamaterial without fiber exhibits random snapping-through instability. (a) Loading–unloading curve. (b) Snapshots from experiments. (c) Snapshots from simulation. Points ① to ⑥ as well as points I to VI in (a) mark the valleys in the loading curves, and the corresponding snapshots and associated nominal strains are given in (b) and (c) for experiments and simulation, respectively. The curved beams marked in purple in (b) and (c) indicate the segments undergoing snapping through instability. Energy dissipation rate in the experiment is measured to be 25.3%.

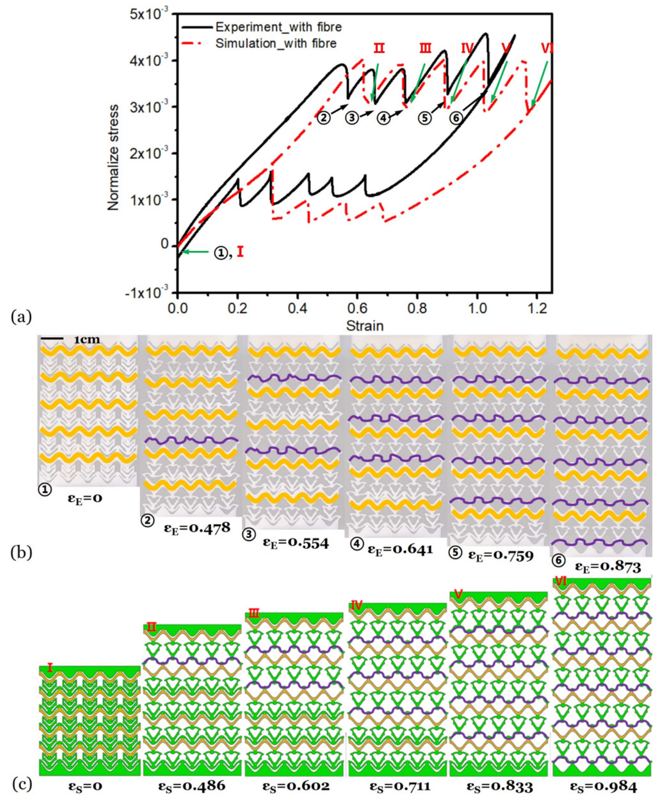


**FIG. 4.** Deformation and sequential snapping-back instability of the mechanical metamaterial reinforced by fibers. (a) Loading–unloading curve of the fiber-reinforced metamaterial. The stress–strain curve exhibits a sequential, layer by layer snapping-back instability behavior, and points ① to ⑥ and I to VI mark, respectively, the valleys of the experiment and simulation curves, where a whole layer of five RVEs snap synchronously. Snapshots of experiments and simulation are presented in (b) and (c), respectively. The orange parts in (b) and (c) represent the hard reinforcing fibers, and the purple curves represent the layers of elements undergoing snapping-back. Energy dissipation rate in the experiment is recorded as 46.4%.

those in the interior of the sheet have different constraints. The boundary layers are fixed, while the interior layers are free to move in the lateral direction. Different boundary conditions interplay with geometric and fabrication imperfections cause different RVEs experience different stress states, resulting in a random occurrence of snapping-through instability. However, the addition of fiber reinforcements significantly increases the rigidity of specified ligaments and thus regulates boundary conditions for all RVEs and modifies the deformation mode from uniaxial tension to pure shear. Therefore, all layers of RVEs, either on the boundary or in interior of the sheet, have identical boundary conditions, and a whole layer of RVEs snap synchronously if the critical strain for onset of instability is reached. Modifying the deformation mode from uniaxial tension to pure shear was previously utilized to enhance electromechanical performance of soft dielectric elastomers,[21,22] but it has never been applied in soft mechanical metamaterials.

The RVE chosen here was previously studied by the authors to attain snapping-back instability based on a RVE analysis.[15] A finite size structure with an array of RVEs, nevertheless, does not demonstrate obvious snapping-back instability as shown in Fig. 3. For the same design but with fiber reinforcement, obvious sequential snapping-back instability is achieved in Fig. 4. This implies that a widely adopted RVE analysis with periodic boundary conditions, a common practice in mechanical metamaterial design, becomes pointless because the RVE behaves loss of representativeness in a finite homogeneous metamaterial, in particular, when large deformation is executed. Our simple yet effective scheme via fiber reinforcement regulates the boundary conditions of any RVE, and the delicately tailored behavior of RVE is still guaranteed in a finite size structure, albeit of large applied strain up to nearly 100%.

There is no snapping or jumping in a displacement-controlled metamaterial sheet undergoing snapping-through instability, and the force decreases, but the displacement increases continuously beyond the force limit point. In contrast to the continuous variation in snapping-through instability, there does exist snapping and sharp drop or increase of force in snapping-back instability. The snapping of structural elements means that more kinetic energy is converted into heat and then dissipated during the snapping-back process. This point is verified by comparing the hysteresis and energy dissipation presented in Figs. 3 and 4, which are obtained by evaluating the area enclosed by the loading–unloading curves. Defining energy dissipation rate as the ratio of the hysteresis to the total area below the loading curve, the experimentally measured energy dissipation rate for metamaterial sheets without and with fiber reinforcement is 25.3% and 46.4%, respectively, both measured up to applied stretch up to 200% and then restored to the original length. Soft metamaterials with fiber reinforcement achieve twice as much as the energy dissipation in those without reinforcement.

The soft mechanical metamaterial sheet reinforced with hard fibers in Fig. 4 shows sequential snapping-back in a layer-by-layer fashion. But the order of sequence, i.e., which layer snaps first when the applied stain reaches the critical strain, is undetermined. We finally theoretically demonstrate that by further introducing small structural variations on purpose into the fiber-reinforced metamaterials, on demand snapping-back is achievable, i.e., a predetermined order of sequence can be selected, which we refer to as "selective" snapping-back instability. Figure 5(a) shows the RVE for a homogeneous metamaterial without fiber. The bold black lines in Fig. 5(a) represent the snapping beams, and small variations in width of beams, increasing from 1.00 mm to 1.08 mm at increment 0.02 mm from bottom to top, are introduced intentionally. These small variations are at first glance indistinguishable from the rest of the structure, but they single out layers snap selectively for the metamaterial with fiber, whereas their effects on metamaterials without fiber is negligible. Figure 5(b) plots the stress–strain curves for metamaterial sheets with various beam widths, black solid line, and red dashed line for sheets without and with fibers, respectively. Figures 5(c) and 5(d) present simulation snapshots, with purple lines for elements undergoing instability and orange parts for fibers. It is clear that the metamaterial without fiber exhibits random instability, irrespective of the introduced artifacts in beam width. Figure 5(d) demonstrates that the fiber reinforced metamaterial with small variations in beam width exhibits snapping-back on demand, with order of sequence of snapping from bottom to top as expected. We also show, in Sec. 2 in the supplementary material, that other orders of sequence of snapping can also be programmed and realized on demand. Through introducing small width variations on purpose for the snapping ligaments, the ligaments with thinner widths tend to snap earlier as they have lower critical load for the onset of instability.

To unveil the underlying mechanics of introducing fibers and varying snapping ligament width, we performed simulations on this aspect and the results are provided in Sec. 3 in the supplementary material. Adding fibers is somehow equivalent to thickening ligament widths, and thus only the latter is considered for the purpose of simplicity. This was done by modeling the metamaterial sheet with the same sizes as given in Fig. 2(a) but with the width of ligament D changed to a serious of values, from 1 mm to 10 mm. Calculating the tensile stiffness of the overall structure and the bending stiffness of a ligament in Fig. S4(c) using the definition of

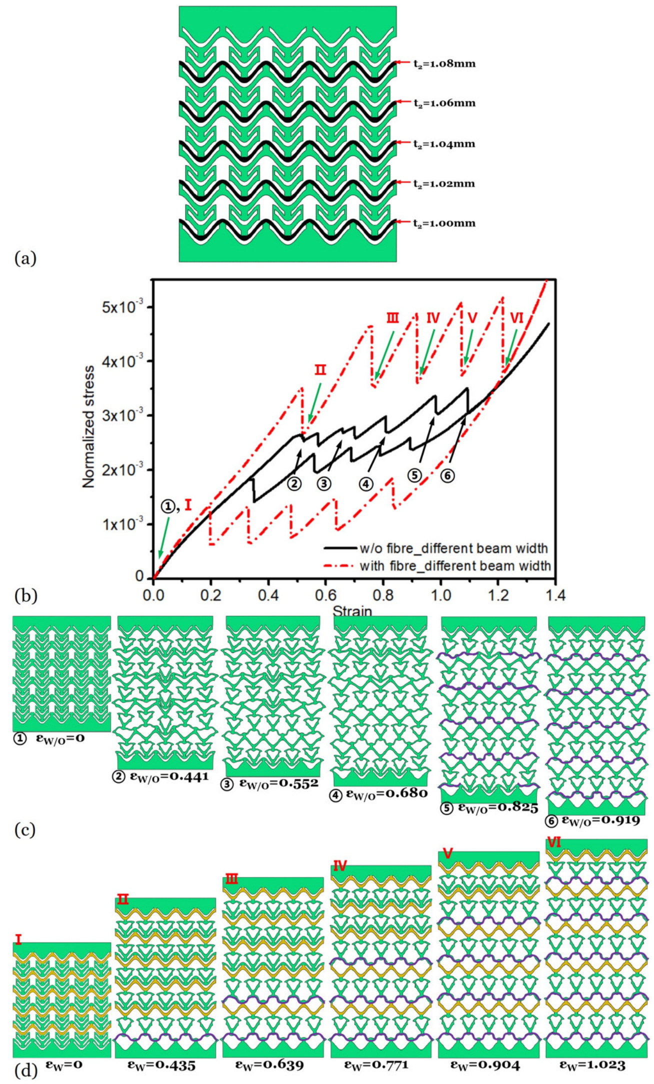


**FIG. 5.** Transition of instability from random snapping-through to selective snapping-back via fiber reinforcement and small structural variations. (a) RVE of a homogeneous metamaterial with variation in beam width. The same RVE was used in (d) but reinforced with fibers which are marked in orange. Both snapshots of the sheet without (c) and with (d) fibers are given for comparison. Instants ①…⑥ and I…VI are marked in the stress–strain curves in (b). The purple lines mark segments undergoing instability.

stiffness given in the supplementary material, in together with the energy dissipation rate defined above, we get the column plots of stiffness and energy dissipation rate curve plotted simultaneously in supplementary material Fig. S4(e). The results indicate that introducing fibers or thickening ligament width has profound effects on the bending stiffness of the ligaments and the variation of energy dissipation rate, but its effects on the tensile stiffness is imperceptible. When the sheet is stretched uniformly, the snapping ligament, however, is in a state of bending. Introducing reinforced-fibers, on the one side, regulates boundary conditions of the snapping ligaments, and on the other side, increases the rigidity of the neighboring ligaments. This local stiffening effect increases the elastic energy stored in the snapping ligaments and causes increase of energy dissipation rate, albeit its effect on increase in the tensile stiffness of the overall structure is negligible.

## IV. CONCLUSION

In summary, we conceive a concept that selective snapping-back instability and enhanced hysteresis can be achieve in soft mechanical metamaterials via fiber reinforcement and small structure variations. The concept is validated through a combination of experiment and numerical simulation. Introducing fiber reinforcement tunes the mechanical behavior of soft mechanical metamaterials from random to programmable and also achieves enhanced hysteresis and energy dissipation. The underlying physics and mechanics behind the phenomenon are elucidated. Our concept provides a simple yet effective way to control and enhance performance of soft mechanical metamaterials.

## SUPPLEMENTARY MATERIAL

See the supplementary material for the material characterization of the rubber and the effects of the ligament width on the instability of fiber-reinforced metamaterial sheets under tension.

## ACKNOWLEDGMENTS

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## DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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