# Modeling Viscoelasticity in Abaqus

Ning An
Xi'an Jiaotong University
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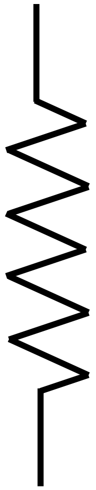
# Spring and dashpot

## Elastic model

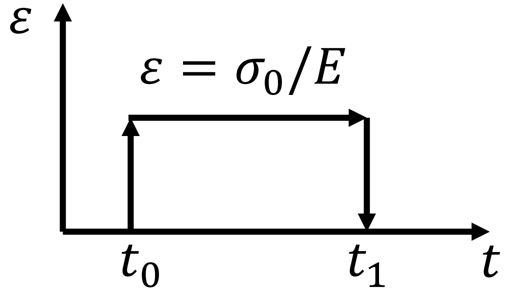


- Elastic materials obey Hooke's law.

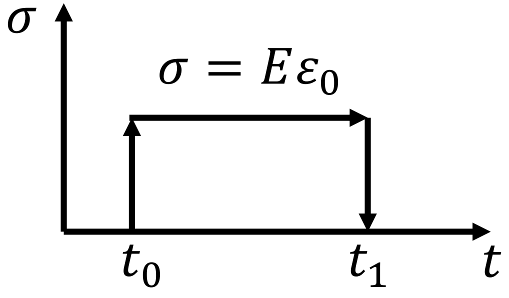
$$\sigma = E\varepsilon$$

- Elastic materials retain their original shape once load is removed.

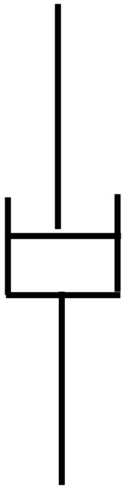
Creep




Stress relaxation




## Viscous model

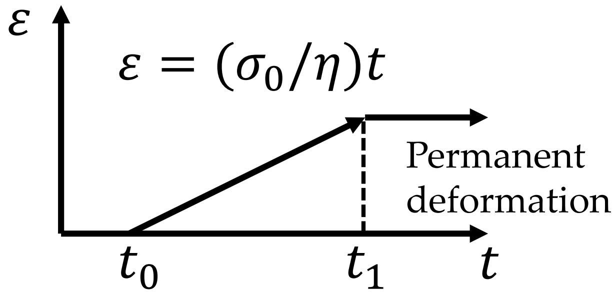


- Elastic materials obey Newton's law of viscosity.

$$\sigma = \eta\dot{\varepsilon}$$

- Viscous materials undergo permanent deformation when the load is removed.

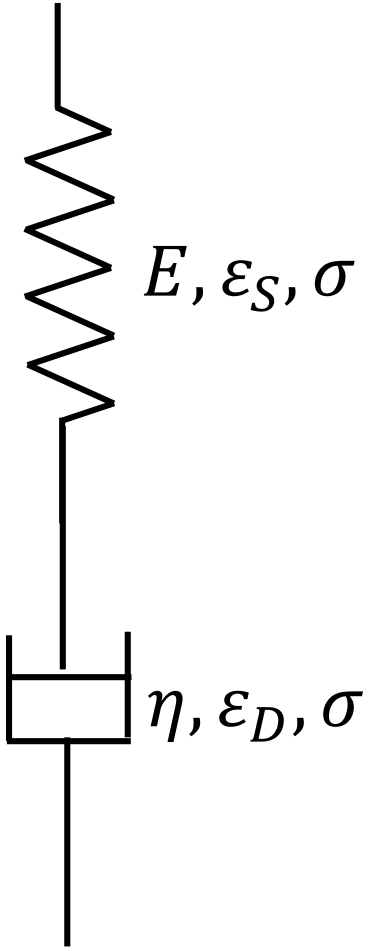
Creep




Stress relaxation

Instantaneous strain is not applicable

# Maxwell model




Time domain

$$\varepsilon(t) = \varepsilon_S(t) + \varepsilon_D(t)$$

$$\sigma(t) = E\varepsilon_S(t)$$

$$\sigma(t) = \eta\dot{\varepsilon}_D(t)$$

Frequency domain

$$\varepsilon(s) = \varepsilon_S(s) + \varepsilon_D(s)$$

$$\sigma(s) = E\varepsilon_S(s)$$

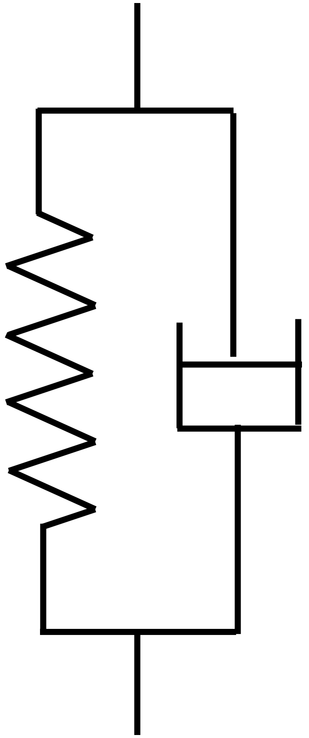
$$\sigma(s) = \eta s\varepsilon_D(s)$$

$$\varepsilon(s) = \frac{\sigma(s)}{E} + \frac{\sigma(s)}{\eta s}$$

$$s\varepsilon(s) = \frac{s\sigma(s)}{E} + \frac{\sigma(s)}{\eta}$$

$$\dot{\varepsilon}(t) = \frac{\dot{\sigma}(t)}{E} + \frac{\sigma(t)}{\eta}$$

# Kelvin-Voigt model




$$\sigma(t) = \sigma_S(t) + \sigma_D(t)$$

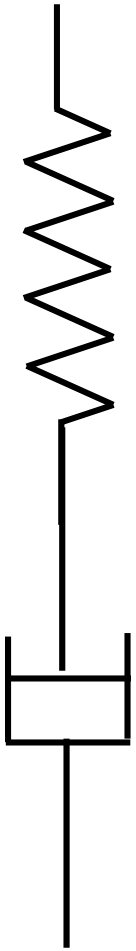
$$\sigma_S(t) = E\varepsilon(t)$$

$$\sigma_D(t) = \eta\varepsilon(\dot{t})$$

$$\sigma(t) = E\varepsilon(t) + \eta\varepsilon(\dot{t})$$

$$\boxed{\varepsilon(\dot{t}) = \frac{\sigma(t)}{\eta} - \frac{E}{\eta}\varepsilon(t)}$$

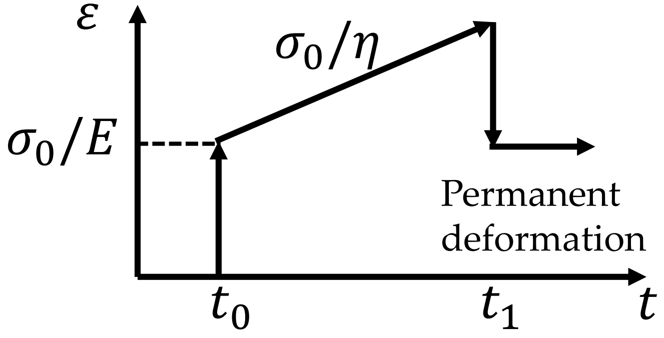
# Basic of visco-elasticity



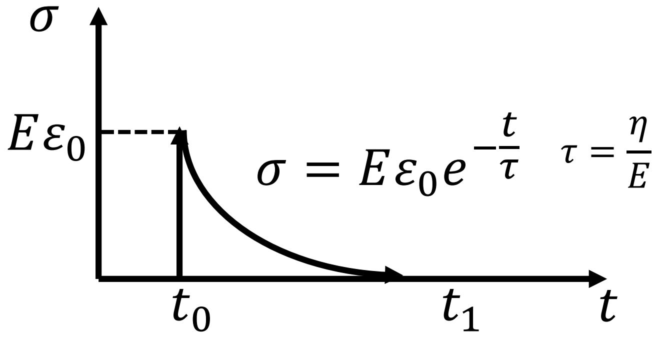
## Maxwell model

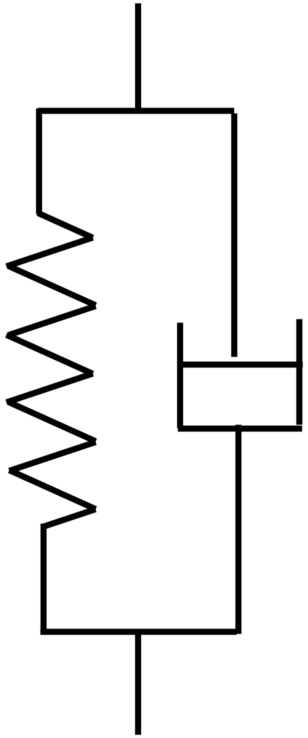
$$\dot{\varepsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$

### Creep




### Stress relaxation

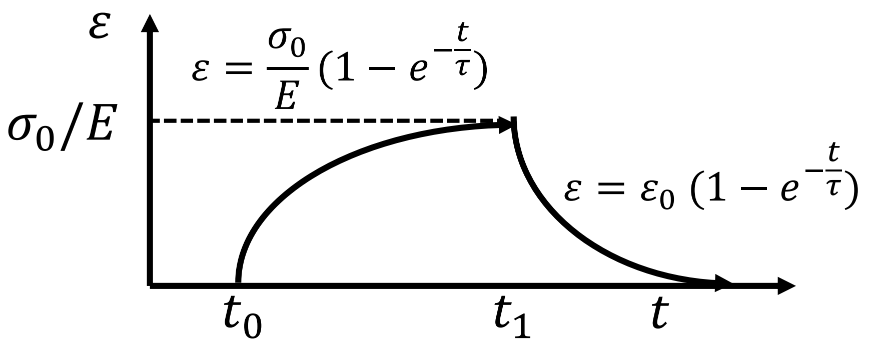


## Viscous model

$$\dot{\varepsilon} = \frac{\sigma}{\eta} - \frac{E}{\eta}\varepsilon$$

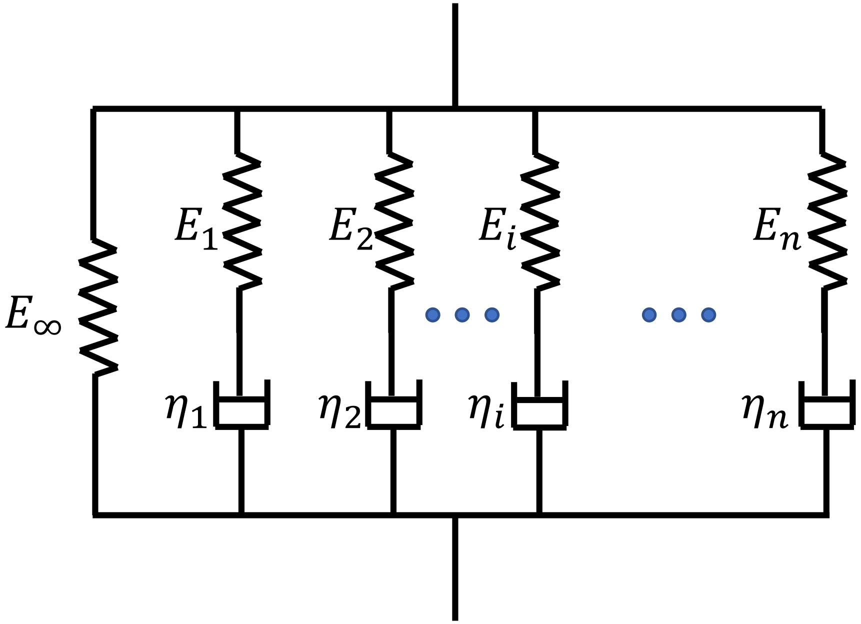
### Creep




### Stress relaxation

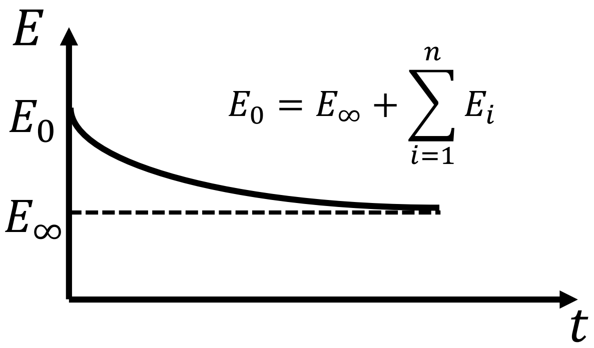
Instantaneous strain is not applicable

# Generalized Maxwell model




$$\sigma(t) = \sigma_\infty(t) + \sum_{i=1}^{n} \sigma_i(t) \qquad \sigma_i(t) = E_i \varepsilon_0 e^{-\frac{t}{\tau_i}}$$

$$\sigma(t) = E_\infty \varepsilon_0 + \sum_{i=1}^{n} E_i \varepsilon_0 e^{-\frac{t}{\tau_i}}$$




Long term modulus form

$$E(t) = E_\infty + \sum_{i=1}^{n} E_i e^{-\frac{t}{\tau_i}}$$

Instantaneous modulus form

$$E(t) = E_0 - \sum_{i=1}^{n} E_i (1 - e^{-\frac{t}{\tau_i}})$$

$$E(t) = E_0 (1 - \sum_{i=1}^{n} m_i (1 - e^{-\frac{t}{\tau_i}})) \quad \text{where } m_i = \frac{E_i}{E_0}$$

# Prony series

Shear modulus

$$G = \frac{E}{2(1+\nu)}$$

Bulk modulus

$$K = \frac{E}{3(1-2\nu)}$$

Young's modulus

$$E(t) = E_0\left(1 - \sum_{i=1}^{n} m_i\left(1 - e^{-\frac{t}{\tau_i}}\right)\right) \quad \text{where } m_i = \frac{E_i}{E_0}$$

Shear modulus

$$G(t) = G_0\left(1 - \sum_{i=1}^{n} g_i\left(1 - e^{-\frac{t}{\tau_i}}\right)\right) \quad \text{where } g_i = \frac{G_i}{G_0}$$

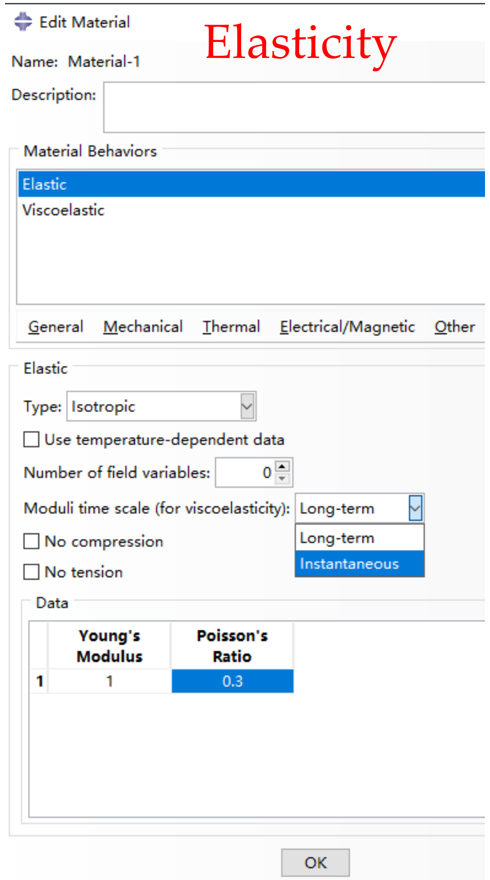
Bulk modulus

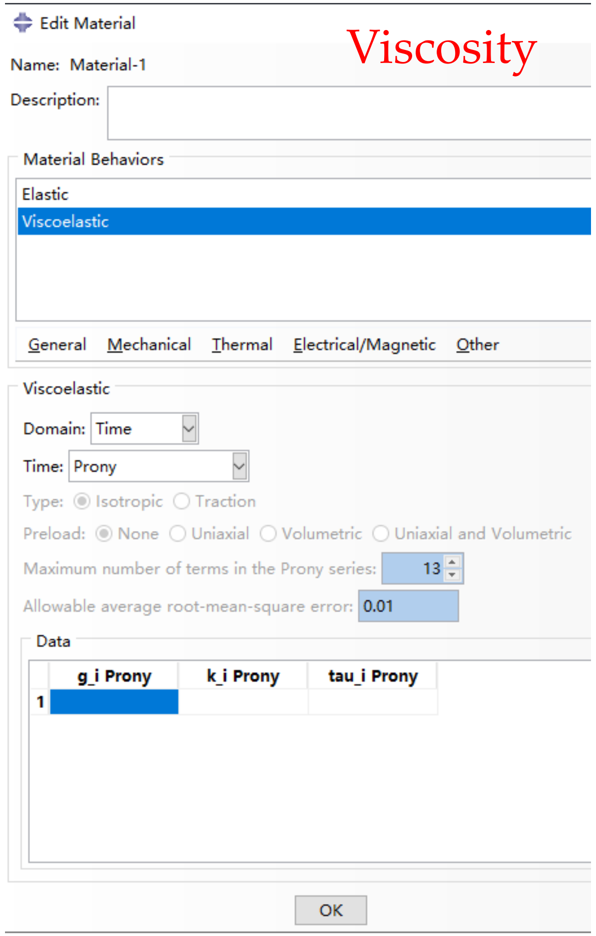
$$K(t) = K_0\left(1 - \sum_{i=1}^{n} k_i\left(1 - e^{-\frac{t}{\tau_i}}\right)\right) \quad \text{where } k_i = \frac{K_i}{K_0}$$

Instantaneous modulus

Relaxation time

Relaxation modulus



# Temperature Effects

Shear modulus

$$G(t) = G_0(1 - \sum_{i=1}^{n} g_i(1 - e^{-\frac{t}{\tau_i \alpha_T}})) \quad \text{where} \;\; g_i = \frac{G_i}{G_0}$$

$\alpha_T$ is the shift factor

The shift function can be defined by the Williams-Landel-Ferry (WLF) approximation, which takes the form:

$$\log \alpha_T = -\frac{c_1(T - T_0)}{c_2 + (T - T_0)}$$

where $T_0$ is the reference temperature at which the relaxation data are given; $T$ is the temperature of interest; and $c_1$, $c_2$ are calibration constants obtained at this temperature. If $T \leq T_0 - c_2$, deformation changes will be elastic, based on the instantaneous moduli.

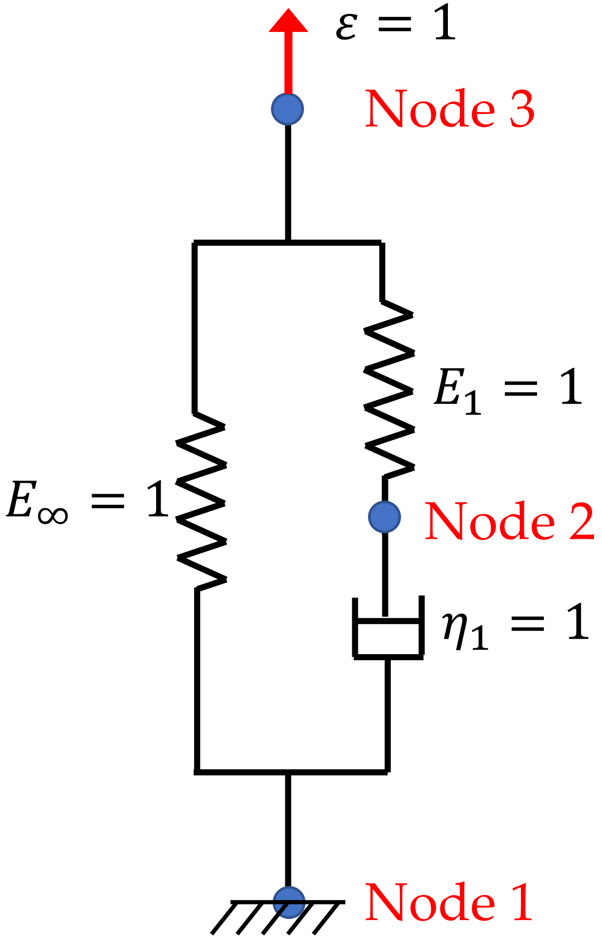
# Standard linear model

https://github.com/Dr-Ning-An/standard-linear-viscoelastic-model

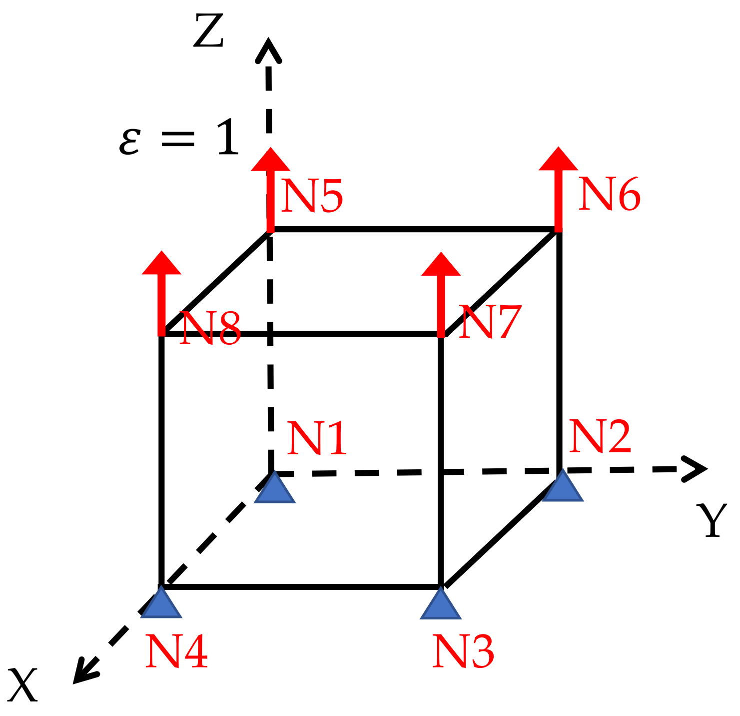
$$\sigma(t) = E_\infty \varepsilon + E_1 \varepsilon e^{-\frac{t}{\tau_1}}$$

Elastic: $E_0 = E_\infty + E_1 = 2 \quad \upsilon = 0.49$

Visco: $g_1 = 0.5 \quad k_1 = 0.5 \quad \tau_1 = 1$

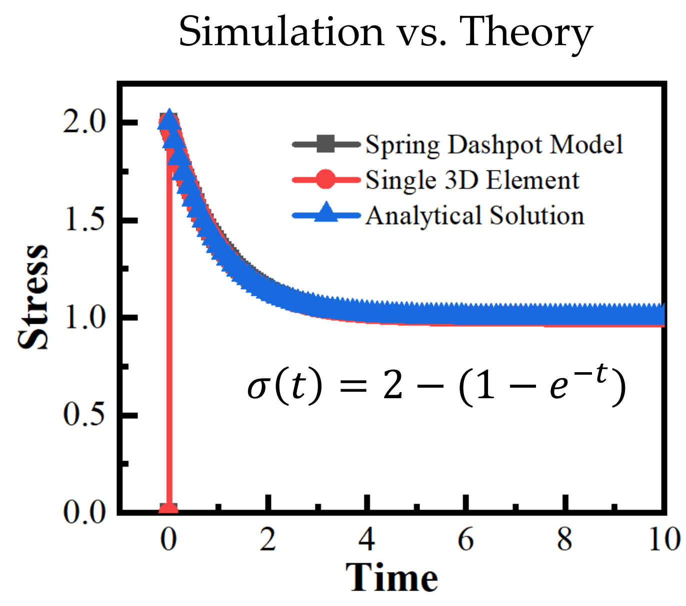



Spring dashpot elements




Single 3D element model

## Results



# Standard linear model with temperature effects

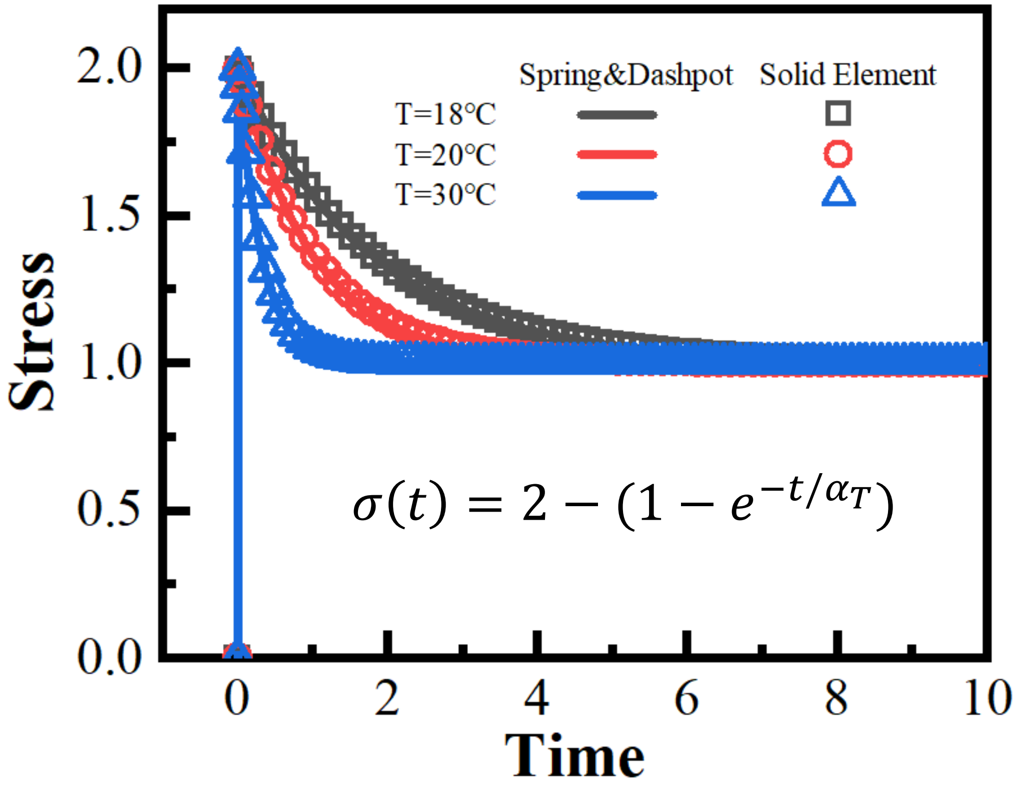
$$\sigma(t) = E_0\varepsilon - E_1\varepsilon(1 - e^{-\frac{t}{\tau_1\alpha_T}})$$

$$\log\alpha_T = -\frac{c_1(T - T_0)}{c_2 + (T - T_0)} \quad T_0 = 20℃ \ c_1 = 1 \ c_2 = 10$$

$$T = 18℃ \ \alpha_T = 1.77827941003892$$

$$T = 20℃ \ \alpha_T = 1.0$$

$$T = 30℃ \ \alpha_T = 0.316227766016838$$




Both time and temperature have a similar effect on the linear viscoelastic properties of polymers.

The long-time relaxation process of a polymer at low temperatures is somehow equivalent to the short-time relaxation process at high temperatures. **Time Temperature Equivalence.**