

Modeling Viscoelasticity in Abaqus

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Spring and dashpot

Elastic model

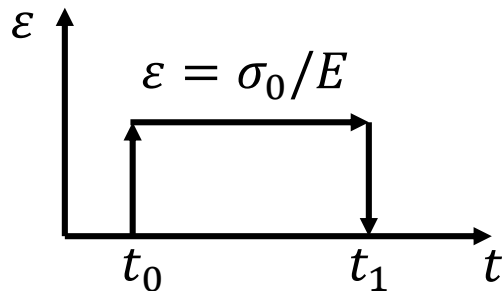


- Elastic materials obey Hooke's law.

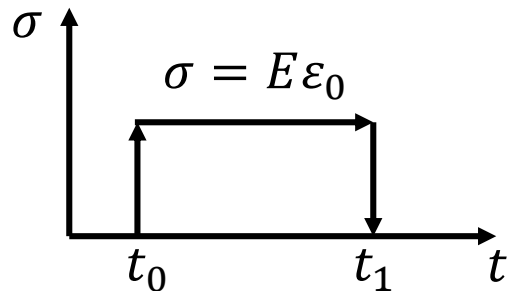
$$\sigma = E\varepsilon$$

- Elastic materials retain their original shape once load is removed.

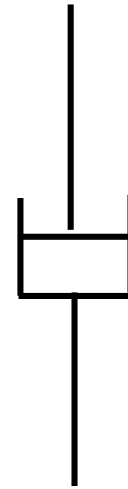
Creep



Stress relaxation



Viscous model

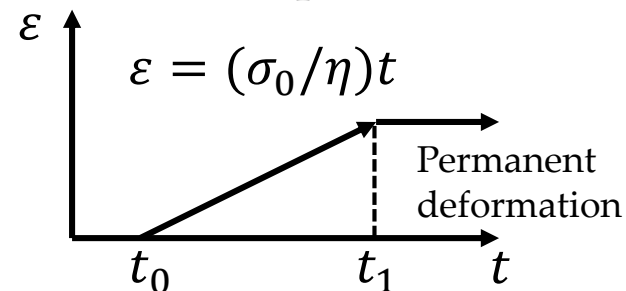


- Elastic materials obey Newton's law of viscosity.

$$\sigma = \eta \dot{\varepsilon}$$

- Viscous materials undergo permanent deformation when the load is removed.

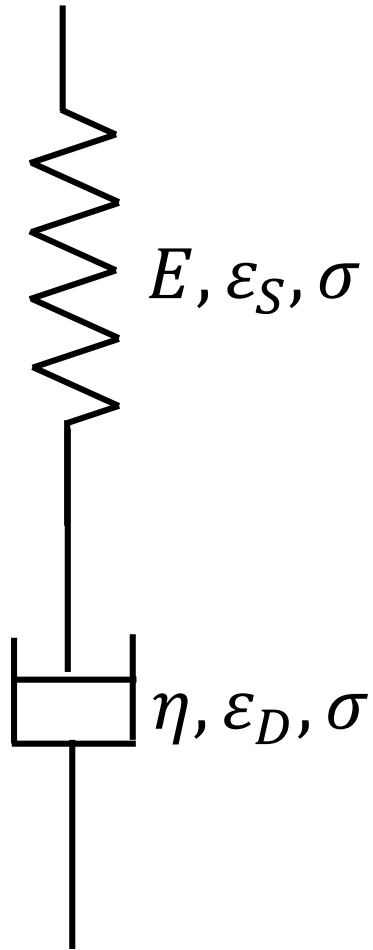
Creep



Stress relaxation

Instantaneous strain is not applicable

Maxwell model



Time domain

$$\varepsilon(t) = \varepsilon_S(t) + \varepsilon_D(t)$$

$$\sigma(t) = E\varepsilon_S(t)$$

$$\sigma(t) = \eta\varepsilon_D'(t)$$

$$\varepsilon'(t) = \frac{\sigma'(t)}{E} + \frac{\sigma(t)}{\eta}$$

Frequency domain

$$\varepsilon(s) = \varepsilon_S(s) + \varepsilon_D(s)$$

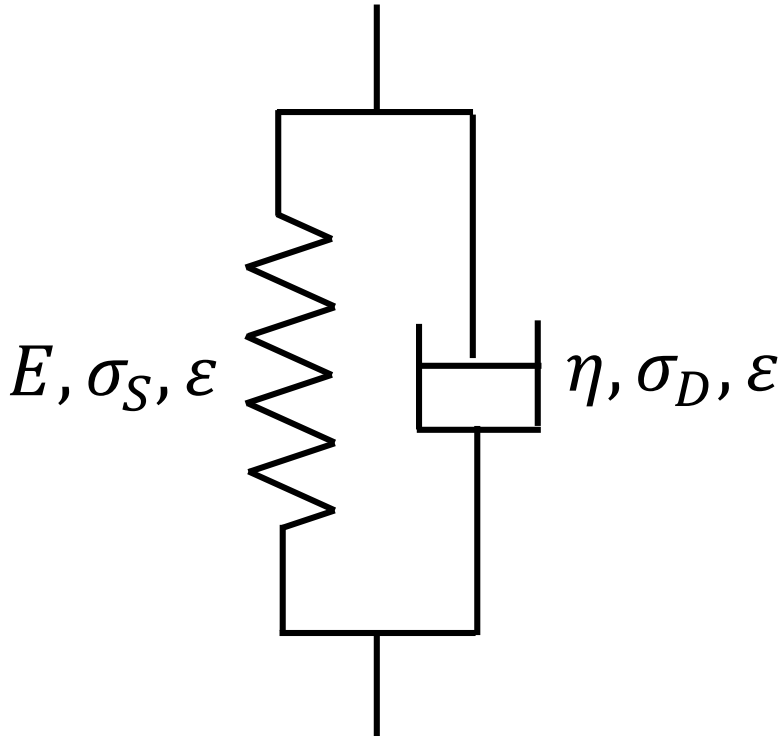
$$\sigma(s) = E\varepsilon_S(s)$$

$$\sigma(s) = \eta s\varepsilon_D(s)$$

$$\varepsilon(s) = \frac{\sigma(s)}{E} + \frac{\sigma(s)}{\eta s}$$

$$s\varepsilon(s) = \frac{s\sigma(s)}{E} + \frac{\sigma(s)}{\eta}$$

Kelvin-Voigt model



$$\sigma(t) = \sigma_S(t) + \sigma_D(t)$$

$$\sigma_S(t) = E\varepsilon(t)$$

$$\sigma_D(t) = \eta\dot{\varepsilon}(t)$$

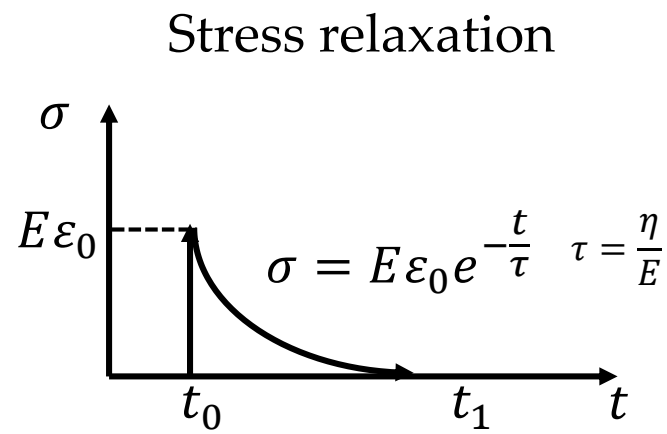
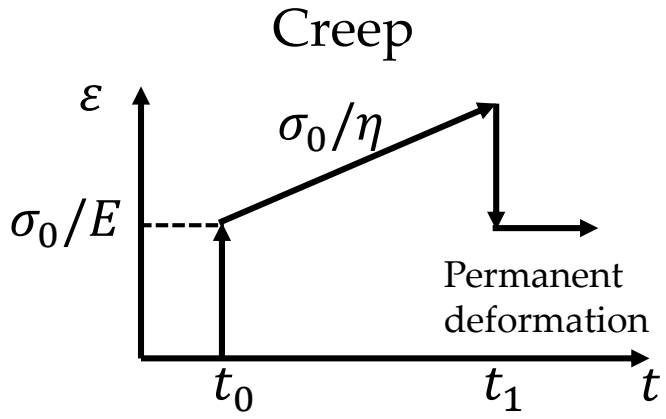
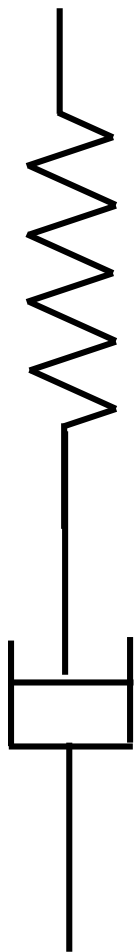
$$\sigma(t) = E\varepsilon(t) + \eta\dot{\varepsilon}(t)$$

$$\dot{\varepsilon}(t) = \frac{\sigma(t)}{\eta} - \frac{E}{\eta}\varepsilon(t)$$

Basic of visco-elasticity

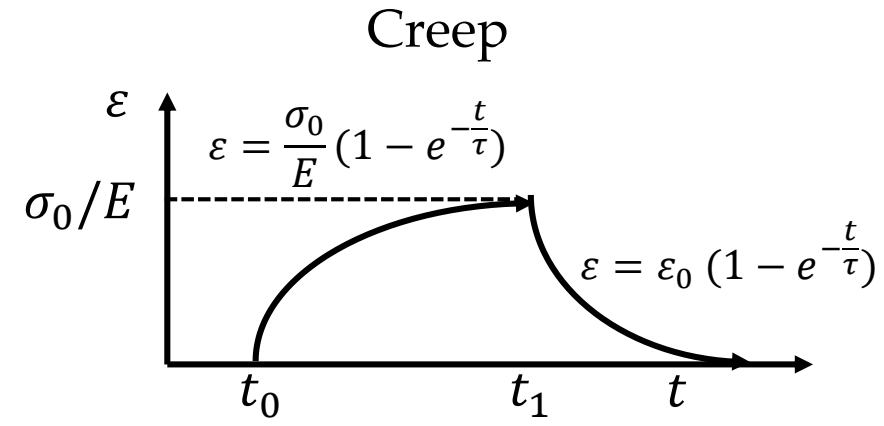
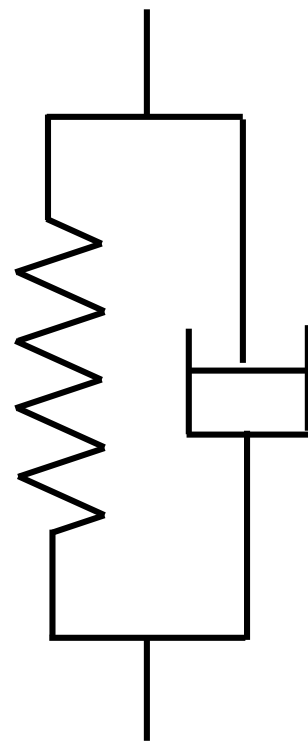
Maxwell model

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\eta}$$



Viscous model

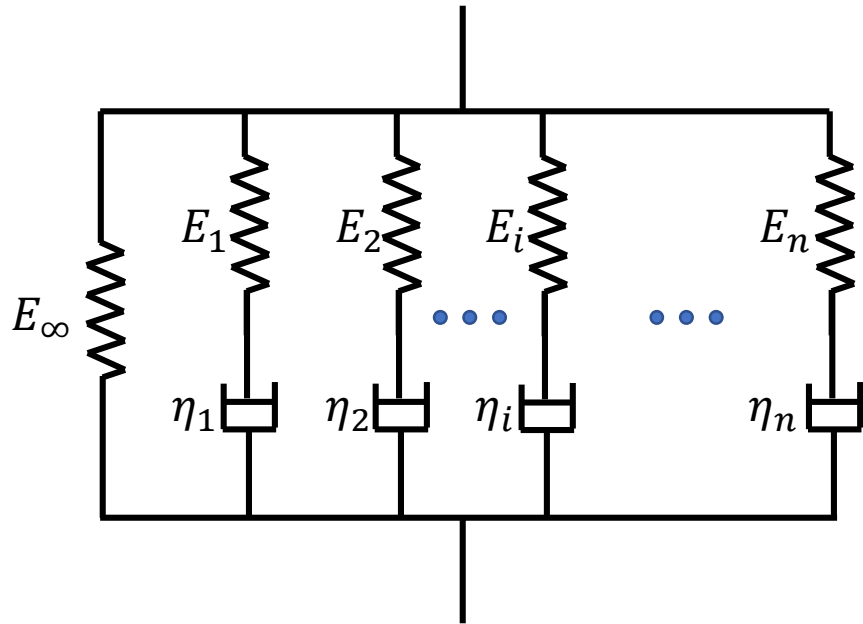
$$\dot{\epsilon} = \frac{\sigma}{\eta} - \frac{E}{\eta} \epsilon$$



Stress relaxation

Instantaneous strain is not applicable

Generalized Maxwell model

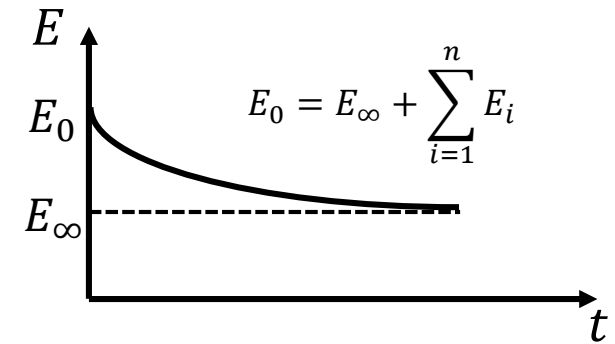


$$\sigma(t) = \sigma_\infty(t) + \sum_{i=1}^n \sigma_i(t) \quad \sigma_i(t) = E_i \varepsilon_0 e^{-\frac{t}{\tau_i}}$$

$$\sigma(t) = E_\infty \varepsilon_0 + \sum_{i=1}^n E_i \varepsilon_0 e^{-\frac{t}{\tau_i}}$$

Long term modulus form

$$E(t) = E_\infty + \sum_{i=1}^n E_i e^{-\frac{t}{\tau_i}}$$



Instantaneous modulus form

$$E(t) = E_0 - \sum_{i=1}^n E_i (1 - e^{-\frac{t}{\tau_i}})$$

$$E(t) = E_0 \left(1 - \sum_{i=1}^n m_i (1 - e^{-\frac{t}{\tau_i}}) \right) \quad \text{where } m_i = \frac{E_i}{E_0}$$

Prony series

Young's modulus

$$E(t) = E_0 \left(1 - \sum_{i=1}^n m_i (1 - e^{-\frac{t}{\tau_i}}) \right) \quad \text{where } m_i = \frac{E_i}{E_0}$$

Shear modulus

$$G(t) = G_0 \left(1 - \sum_{i=1}^n g_i (1 - e^{-\frac{t}{\tau_i}}) \right) \quad \text{where } g_i = \frac{G_i}{G_0}$$

Bulk modulus

$$K(t) = K_0 \left(1 - \sum_{i=1}^n k_i (1 - e^{-\frac{t}{\tau_i}}) \right) \quad \text{where } k_i = \frac{K_i}{K_0}$$

Instantaneous modulus

Relaxation modulus

Relaxation time

Shear modulus

$$G = \frac{E}{2(1 + \nu)}$$

Bulk modulus

$$K = \frac{E}{3(1 - 2\nu)}$$

Edit Material

Name: Material-1

Description:

Material Behaviors

- Elastic
- Viscoelastic

General Mechanical Thermal Electrical/Magnetic Other

Elastic

Type: Isotropic

Use temperature-dependent data

Number of field variables: 0

Moduli time scale (for viscoelasticity): Long-term

No compression

No tension

Data

	Young's Modulus	Poisson's Ratio
1	1	0.3

OK

Elasticity

Edit Material

Name: Material-1

Description:

Material Behaviors

- Elastic
- Viscoelastic

General Mechanical Thermal Electrical/Magnetic Other

Viscoelastic

Domain: Time

Time: Prony

Type: Isotropic Traction

Preload: None Uniaxial Volumetric Uniaxial and Volumetric

Maximum number of terms in the Prony series: 13

Allowable average root-mean-square error: 0.01

Data

	g_i Prony	k_i Prony	tau_i Prony
1			

OK

Viscosity

Temperature Effects

Shear modulus

$$G(t) = G_0 \left(1 - \sum_{i=1}^n g_i (1 - e^{-\frac{t}{\tau_i \alpha_T}}) \right) \quad \text{where } g_i = \frac{G_i}{G_0}$$

α_T is the shift factor

The shift function can be defined by the Williams-Landel-Ferry (WLF) approximation, which takes the form:

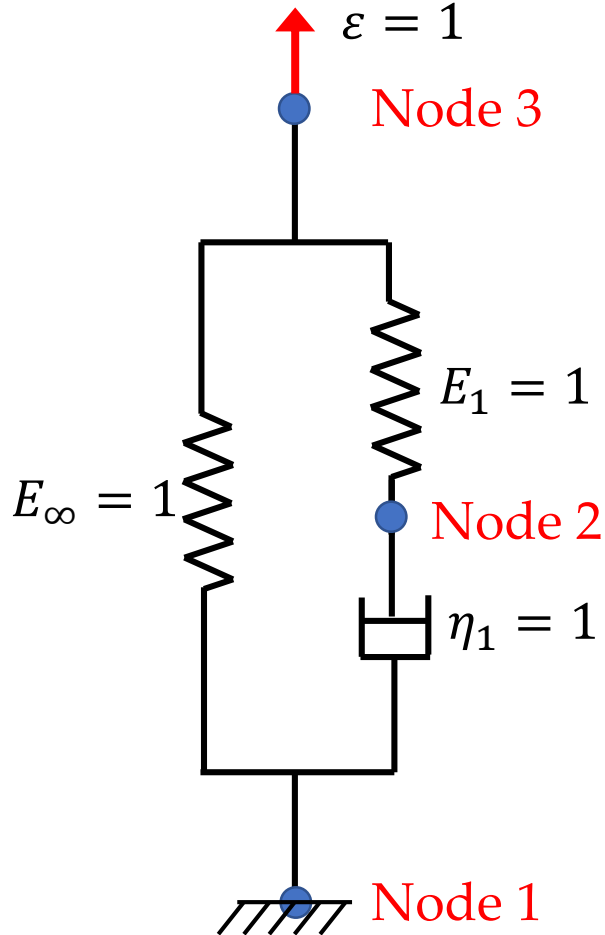
$$\log \alpha_T = - \frac{c_1 (T - T_0)}{c_2 + (T - T_0)}$$

where T_0 is the reference temperature at which the relaxation data are given; T is the temperature of interest; and c_1, c_2 are calibration constants obtained at this temperature. If $T \leq T_0 - c_2$, deformation changes will be elastic, based on the instantaneous moduli.

Standard linear model

<https://github.com/Dr-Ning-An/standard-linear-viscoelastic-model>

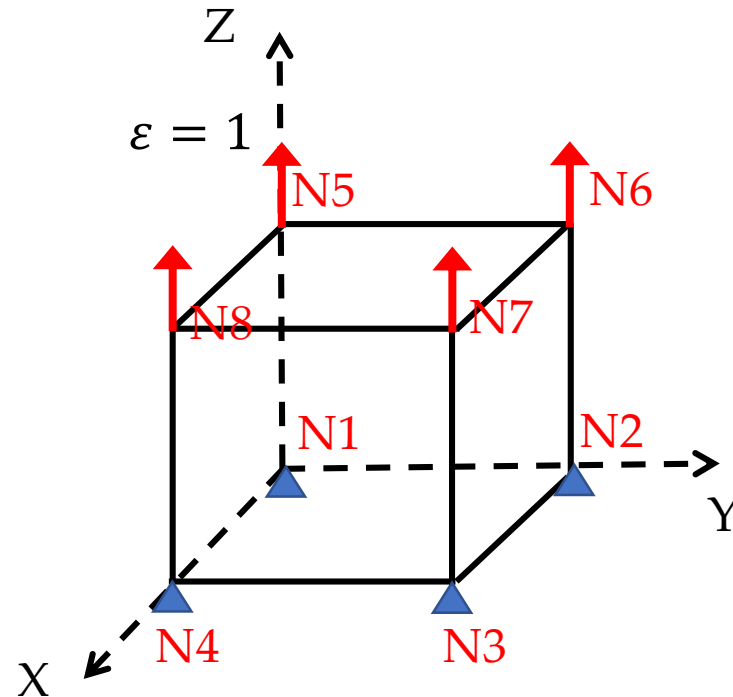
$$\sigma(t) = E_{\infty}\varepsilon + E_1\varepsilon e^{-\frac{t}{\tau_1}}$$



Spring dashpot elements

Elastic: $E_0 = E_{\infty} + E_1 = 2$ $\nu = 0.49$

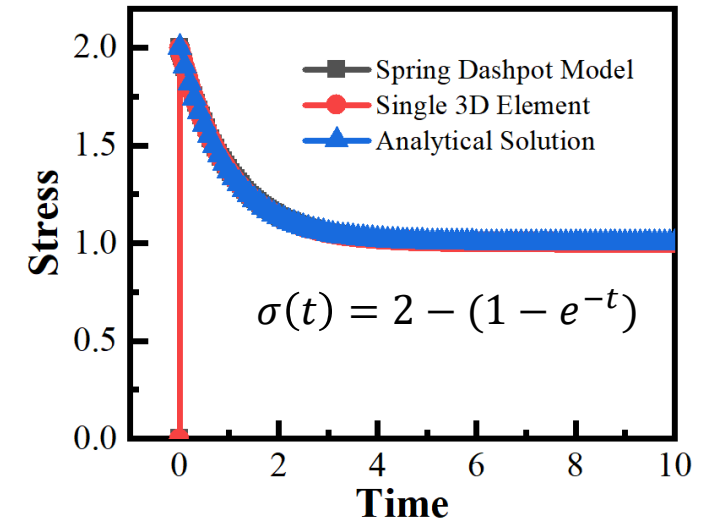
Visco: $g_1 = 0.5$ $k_1 = 0.5$ $\tau_1 = 1$



Single 3D element model

Results

Simulation vs. Theory



Standard linear model with temperature effects

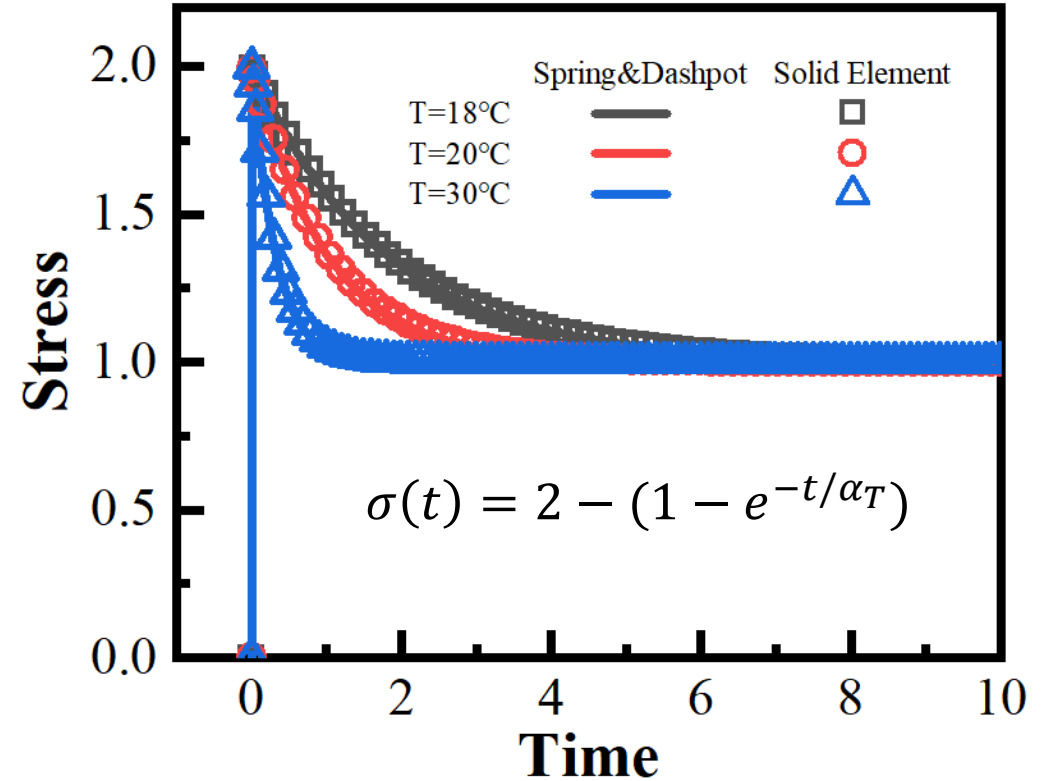
$$\sigma(t) = E_0 \varepsilon - E_1 \varepsilon (1 - e^{-\frac{t}{\tau_1 \alpha_T}})$$

$$\log \alpha_T = -\frac{c_1(T - T_0)}{c_2 + (T - T_0)} \quad T_0 = 20^\circ\text{C} \quad c_1 = 1 \quad c_2 = 10$$

$$T = 18^\circ\text{C} \quad \alpha_T = 1.77827941003892$$

$$T = 20^\circ\text{C} \quad \alpha_T = 1.0$$

$$T = 30^\circ\text{C} \quad \alpha_T = 0.316227766016838$$



Both time and temperature have a similar effect on the linear viscoelastic properties of polymers.

The long-time relaxation process of a polymer at low temperatures is somehow equivalent to the short-time relaxation process at high temperatures. **Time Temperature Equivalence.**